Math 2211: Practice Final

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(1) Problems from Chapter 12.

(a) Find the value of x such that the vectors $\langle 3, 2, x\rangle$ and $\langle 2x, 4, x\rangle$ are orthogonal.

(b) Find two unit vectors that are orthogonal to both $\langle 0, 1, 2\rangle$ and $\langle 1, -2, 3\rangle$.

(c) Find the equation of the plane passing through the points $(3, -1, 1), (4, 0, 2)$ and $(6, 3, 1)$.

(d) Find the equation of the plane passing through $(1, 2, -2)$ that contains the line $x = 2t$, $y = 3 - t$, $z = 1 + 3t$.

(e) Find the point in which the line with parametric equations $x = 2 - t$, $y = 1 + 3t$ and $z = 4t$ intersects the plane $2x - y + z = 2$.

(f) Find the angle between the planes $x + y - z = 1$ and $2x - 3y + 4z = 5$.

(g) Find the distance between the planes $3x + y - 4z = 2$ and $3x + y - 4z = 24$,

(h) Find an equation of the plane containing the line of intersection of the planes $x - z = 1$, $y + 2z = 3$ and perpendicular to the plane $x + y - 2z = 1$.

(2) Problems from Chapter 13.

(a) Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$.

(b) Let $\mathbf{r}(t) = \langle \sqrt{2-t}, \frac{e^t - 1}{t}, \ln(t+1) \rangle$. Find the domain of $\mathbf{r}(t)$. Find $\lim_{t \to 0} \mathbf{r}(t)$. Compute $\mathbf{r}'(t)$.

(c) Find the length of the curve $\mathbf{r}(t) = \langle 2t^{3/2}, \cos(2t), \sin(2t) \rangle$ where $0 \leq t \leq 1$.

(d) Find the vector $\mathbf{T}, \mathbf{N}$ and $\mathbf{B}$ for the curve $\mathbf{r}(t) = \langle t^2, \frac{2}{3}t^3, t \rangle$ at the point $\langle 1, \frac{2}{3}, 1 \rangle$

(e) For the curve given by $\mathbf{r}(t) = \langle \frac{1}{6}t^3, \frac{1}{2}t^2, t \rangle$, find the unit tangent vector, the unit normal vector and the curvature.

(f) At what point on the curve $x = t^3$, $y = 3t$ and $z = t^4$ is the normal plane parallel to the plane $6x + 6y - 8z = 1$.

(g) Find the velocity and position vectors of a particle that has acceleration $\mathbf{a}(t) = \langle 2, 6t, 12t^2 \rangle$ with $\mathbf{v}(0) = \langle 1, 0, 0 \rangle$ and $\mathbf{r}(0) = \langle 0, 1, -1 \rangle$.

(h) Find the tangential and normal components of the acceleration vector of $\mathbf{r}(t) = \langle (3t - t^3), 3t^2 \rangle$.

(3) Problems from Chapter 14.

(a) Evaluate the limit or show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^3}$$

(b) If $z = xy + xe^{\frac{y}{x}}$, then show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy + z.$$

(c) Find the equation of the tangent plane and the normal line to the given surface $\sin(xyz) = x + 2y + 3z$ at the point $(2, -1, 0)$.

(d) Find the points on the hyperboloid $x^2 + 4y^2 - z^2 = 4$ where the tangent plane is parallel to the plane $2x + 2y + z = 5$.

(e) Find du if $u(s,t) = \ln(1 + se^{2t})$.

(f) If $u = x^2y^3 + z^4$, where $x = p + 3p^2$, $y = pe^p$ and $z = p\sin p$, then find $\frac{du}{dp}$ using the chain rule.

(g) If $z = y + f(x^2 - y^2)$, where f is a differentiable function. Show that

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x.$$

(h) Find the directional derivative of $f = x^2y + x\sqrt{1+z}$, at the point $(1, 2, 3)$ in the direction $\langle 2, 1 - 2\rangle$. Furthermore, find the maximum rate of change at the point $(1, 2, 3)$ and specify the direction where it occurs.

(i) Find the local maximum and minimum values and saddle points of the function $f(x, y) = (x^2 + y)e^{\frac{y}{2}}$ in $\mathbb{R}^2$.

(j) Find the absolute maximum and minimum values of $f = e^{-x^2-y^2}(x^2 + 2y^2)$ on the disk D defined by $x^2 + y^2 \leq 4$.

(k) Use Lagrange multipliers to find the maximum and minimum values of $f = \frac{1}{x} + \frac{1}{y}$ subject to the constraint $\frac{1}{x^2} + \frac{1}{y^2} = 1$.

(4) Problems from Chapter 15:

(a) Evaluate

$$\int_0^{\pi}\int_0^1\int_0^{\sqrt{1-y^2}} y\sin(x)\,dzdydx.$$

(b) Describe the solid whose volume is given by the integral below and evaluate the integral.

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \rho^2\sin(\phi)\,d\rho d\phi d\theta.$$

(c) Evaluate

$$\int_0^1\int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3}dxdy.$$

(d) Let D be the region in the first quadrant that lies above the hyperbola $xy = 1$ and the line $y = x$ and below the line $y = 2$. Evaluate

$$\iint_D y\,dA.$$

(e) Let D be the region in the first quadrant bounded by the lines $y = 0$ and $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$. Evaluate

$$\iint_D (x^2 + y^2)^{3/2}\,dA.$$

(f) Let H be the solid hemisphere that lies above the xy-plane and has center the origin and radius 1. Evaluate

$$\iiint_H z^3\sqrt{x^2+y^2+z^2}dV.$$

(g) Find the volume of the solid that lies above the paraboloid $z = x^2 + y^2$ and below the half-cone $z = \sqrt{x^2+y^2}$.

(h) Evaluate using spherical coordinates

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dzdxdy$$

(i) Evaluate $\iint_R (4x+8y)dA$ where R is the parallelogram with vertices $(-1,3), (1,-3), (3,-1)$ and $(1,5)$ using the transformation $x = \frac{1}{4}(u+v)$ and $y = \frac{1}{4}(v-3u)$.

(5) Problems from Chapter 16.

(a) Evaluate the line integral $\int_C y\,ds$ where $C : x = 2\sin t,\ y = t$ and $z = -2\cos t,\ 0 \le t \le 1$.

(b) Evaluate $\int_C (x + yz)\,dx + 2x\,dy + xyz\,dz$ where C consists of the line segments from $(1, 0, 1)$ to $(2, 3, 1)$ and $(2, 3, 1)$ to $(5, 5, 5)$.

(c) Evaluate $\int_C \sin x\,dx + \cos y\,dy$ where C consists of the top half of the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$ and the line segment from $(-1, 0)$ to $(-2, 3)$.

(d) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle \sin x, \cos y, xz\rangle$ and $\mathbf{r}(t) = \langle t^3, -t^2, t\rangle$ with $0 \le t \le 1$.

(e) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}\rangle$ and C is the parabola $y = 1+x^2$ from $(-1, 2)$ to $(1, 2)$.

(f) Find a function f such that $\mathbf{F} = \nabla f$ and use it to calculate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle e^y,\, xe^y,\, (z+1)e^z \rangle$, $C:\ \mathbf{r}(t) = \langle t, t^2, t^3 \rangle$, $0 \le t \le 1$.

(g) Show that the line integral is independent of path and evaluate the integral.

$$\int_C (1 - ye^{-x})\, dx + e^{-x}\, dy, \quad C \text{ is any path from } (0,1) \text{ to } (1,2)$$

(h) Verify that Green's Theorem is true for the line integral $\int_C xy^2\, dx - x^2 y\, dy$ where C consist of the parabola $y = x^2$ from $(-1,1)$ to $(1,1)$ and the line segment from $(1,1)$ to $(-1,1)$.

(i) Use Green's Theorem to evaluate $\int_C \sqrt{1+x^2}\, dx + 2xy\, dy$ where C is the triangle with vertices $(0,0), (1,0), (1,3)$.

(j) Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle \sqrt{x} + y^3,\, x^2 + \sqrt{y} \rangle$ where C consists of arc of the curve $y = \sin x$ from $(0,0)$ to $(\pi,0)$ and the line segment from $(\pi,0)$ to $(0,0)$.