Math 2211: Practice Midterm 1

Calculus 3

(1) Let $\vec{u} = \langle -2, 3, 6 \rangle$, $\vec{u} = \langle -7, 1, 4 \rangle$ and $\vec{w} = \langle 3, 5, 2 \rangle$.

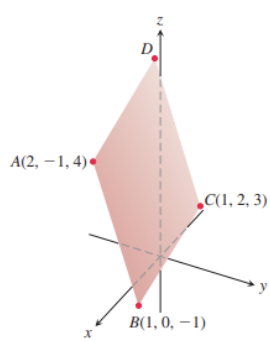
(a) Find $2\vec{u} - 3\vec{v} + \vec{w}$, $\vec{u} \cdot \vec{w}$, and $\vec{u} \times \vec{v}$.

(b) Find the area of the parallelogram spanned by $\vec{u}$ and $\vec{v}$.

(c) Find $\vec{u} \cdot (\vec{v} \times \vec{w})$.

(d) Find $proj_{\vec{u}}\, \vec{v}$ and $comp_{\vec{u}}\, \vec{v}$.

(e) The parallelogram shown here has vertices at $A(2, -1, 4)$, $B(1, 0, -1)$, $C(1, 2, 3)$, and D. Find the coordinate D.



(2) Solve the following problems:

(a) Find the parametric equation of the line passing through the two points $P(4,5,2)$ and $Q(-2,-1,7)$.

(b) Find the parametric equation of the line passing through the point $P(1,5,2)$ parallel to the vector $\vec{v} = \langle 2,3,2\rangle$.

(c) Find the equation of the plane passing through the points

$$P(1,5,2), \quad Q(10,4,-1), \quad \text{and} \quad R(-1,2,-1).$$

(d) Find an equation for the plane that passes through the point $(3,-2,1)$ normal to the vector $\vec{n} = 2\vec{i} + \vec{j} + 20\vec{k}$.

(e) Find an equation for the plane that passes through the point $(-1,6,0)$ and is perpendicular to the line $x = -1+t, y = 6-2t, z = 3t$.

(f) Find parametric equations for the line in which the planes $3x + 6z = 1$ and $2x + 2y - z = 3$ intersect. Show that the plane are orthogonal to each other.

(3) Solve the following problems:

(a) Find the set of points that are equidistant from the points $P(1, 0, 4)$ and $Q(6, 2, -1)$.

(b) Find the distance between the following parallel planes:

$$3x - 2y + z = 2 \quad \text{and} \quad 3x - 2y + z = 7.$$

(c) Find the distance from the point $(2, 2, 3)$ to the plane $2x + 3y + 5z = 0$.

(4) Solve the following:

(a) If $\mathbf{r}(t) = \langle e^{2t}, e^{-5t}, t\rangle$, find $\mathbf{r}'(0), \mathbf{r}''(0)$, $\mathbf{r}'(0) \times \mathbf{r}''(0)$ and $\mathbf{T}(0) = \frac{\mathbf{r}'(0)}{|\mathbf{r}'(0)|}$. Also find the curvature at point $t = 0$.

(b) Find $f'(3)$, where $f(t) = \mathbf{u}(t) \cdot \mathbf{v}(t)$, $\mathbf{u}(3) = \langle 1, 3, -3\rangle$, $\mathbf{u}'(3) = \langle 9, 1, 7\rangle$ and $\mathbf{v}(t) = \langle t^2, t, t^3\rangle$.

(c) Suppose $\vec{r}(t) = (e^t \cos t)\vec{i} + (e^t \sin t)\vec{j}$. Show that the angle between $\vec{r}$ and $\vec{a}$ never changes. What is the angle?

(d) Find the lengths of the curve

$$\vec{r}(t) = (2\cos t)\vec{i} + (2\sin t)\vec{j} + (t^2)\vec{k}, \quad 0 \le t \le \pi/4.$$

(e) Find equations for the osculating, normal, and rectifying planes of the curve $r(t) = \langle t, t^2, t^3\rangle$ at the point $(1, 1, 1)$.

(f) Find parametric equations for the line that is tangent to the curve

$$r(t) = \langle e^t, \sin t, \ln(1-t)\rangle, \quad \text{at the point } t = 0.$$

(5) The graph of a function $f(t)$ in the xy-plane is the curve

$$\mathbf{r}(t) = \langle t, f(t), 0 \rangle.$$

(a) Assuming that f has first and second order derivatives, compute the curvature of this curve at a point $\mathbf{r}(t)$ as a function of $f'(t)$ and $f''(t)$.
Hint: the curvature is given by $\kappa = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$.

(b) In the special case $f(t) = t^2$, compute the curvature at the point $t = 0$ and the coordinates of the center of the osculating circle at this point.

(c) Draw a graph that confirms these values.