# Math 2211: Practice Midterm 2

Calculus 3

(1) Solve the following problems.

(a) Find and sketch the domain of $f(x, y, z) = e^{\sqrt{z-x^2-y^2}}$. Find the range of $f$.

(b) Find the domain and range of $f(x, y) = \arcsin(x^2 + y^2 - 2)$.

(c) Draw a contour map of $f(x, y) = x^3 - y$.

(d) .

Match the function with its graph (labeled I–VI).Give reasons for your choices.

(a) $f(x, y) = |x| + |y|$

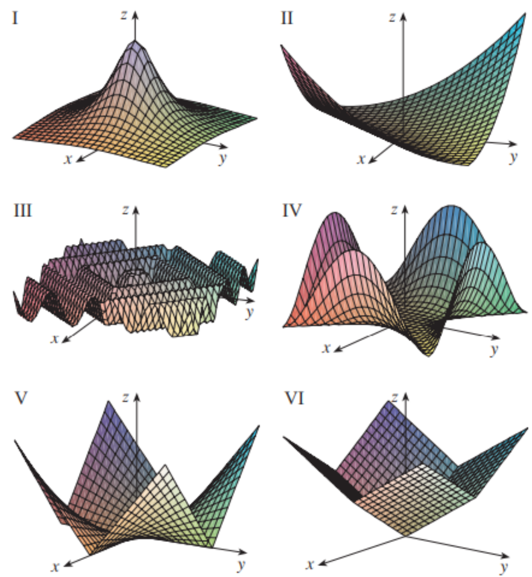
(b) $f(x, y) = |xy|$

(c) $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$

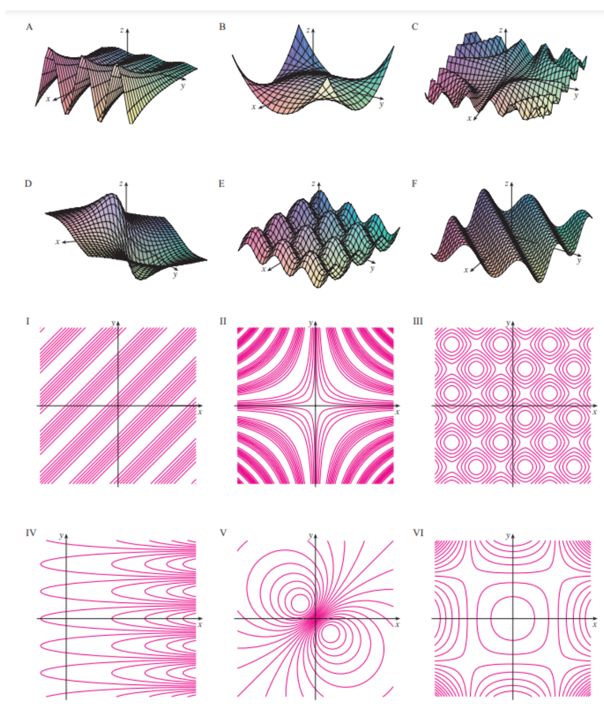
(d) $f(x, y) = (x^2 - y^2)^2$

(e) $f(x, y) = (x - y)^2$

(f) $f(x, y) = \sin(|x| + |y|)$




(e) .



(2) Solve the following problems:

(a) Find the limit

$$\lim_{(x,y)->(0,0)} \frac{2x^2y}{x^2+y^2}.$$

(b) Find the limit

$$\lim_{(x,y)->(0,0)} \frac{y^4}{x^4+3y^4}.$$

(c) Find the limit

$$\lim_{(x,y)->(0,0)} \frac{x^4-y^4}{x^2+y^2}.$$

(d) Find the limit

$$\lim_{(x,y)->(3,0,1)} e^{-xy}\sin\left(\frac{\pi z}{2}\right).$$

(e) Find the limit

$$\lim_{(x,y)->(0,0)} \frac{x^2+\sin^2(y)}{2x^2+y^2}.$$

(f) Find the limit

$$\lim_{(x,y)->(0,0)} \frac{2xy^4}{3x^2+y^8}.$$

(3) Solve the following problems:

(a) Find $f_x$ and $f_y$ of the function $f(x,y) = \int_y^x \cos(t^2)dt$.

(b) Find $f_x, f_y$ and $f_z$ of the function $f(x,y) = xz - 5x^2y^3z^4$.

(c) Find $f_x(3,4)$ where $f(x,y) = \ln(x + \sqrt{x^2 + xy^2})$.

(d) Find the tangent plane of $f(x,y) = \sqrt{x + e^{4y} + xz}$ at the point $(1,0,8)$.

(e) Verify the linear approximation $\sqrt{y + \cos^2 x} \approx 1 + \frac{1}{2}y$ at the point (0,0).

(f) Verify the linear approximation $\frac{2x+3}{4y+1} \approx 3 + 2x - 12y$ at the point (0,0).

(4) Solve the following:

(a) If $z = e^x \sin(y)$, where $x = st^2$ and $y = s^2 t$, find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

(b) Use chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ if $z = e^r \cos(\theta)$, $r = st$ and $\theta = \sqrt{s^2 + 5t}$.

(c) If $z = f(x, y)$, where $x = r\cos(\theta)$ and $y = r\sin(\theta)$ then find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$. Use them to show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

(d) Find $\frac{dy}{dx}$ of the following equation

$$\sqrt{xy} = 1 + x^2 y.$$

(e) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of the following equation

$$x^2 - x^3 = \arctan(yz).$$

(5) Solve the following:

(a) Given that $f(x,y) = \frac{y^2}{x}$. Find the gradient of $f$. Evaluate the gradient at the point $P(1,2)$. Find the rate of change of at in the direction of the vector $\vec{u} = \frac{1}{3}(2\vec{i} + \sqrt{5}\vec{j})$.

(b) Find the directional derivative of the function $f(x,y) = (x + 2y + 3z)^{3/2}$ at the point $(1,2,1)$ in the direction of the vector $\langle 0, 2, -1 \rangle$.

(c) Find the maximum rate of change of $f(x,y) = \sin xy$ at the point $(1,0)$ and the direction in which it occurs.

(d) Show that the equation of the tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at the point $(x_0, y_0, z_0)$ can be written as

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = 1.$$

(e) Two surfaces are called orthogonal at a point of intersection if their normal lines are perpendicular at that point. Show that surfaces with equations $F(x,y,z) = 0$ and $G(x,y,z) = 0$ are orthogonal at a point $P$ where $\Delta F \neq 0$ and $\Delta G \neq 0$ if and only if

$$F_x G_x + F_y G_y + F_z G_z = 0 \quad \text{at } P.$$