Math 2211: Recitation 7 (T)

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(1) Solve any **two** the following problems:

(a) Use the Chain Rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial s}$ of

$$z = \arcsin(x - y), \quad x = s^2 + t^2, \quad y = 1 - 2st.$$

Hint: $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$.

(b) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of the equation

$$yz = \ln(x + z).$$

(c) Let $W(s,t) = F(u(s,t), v(s,t))$, where F, u and v are differentiable, and

$$\begin{aligned} u(1,0) &= 2, & v(1,0) &= 3, \\ u_s(1,0) &= -2, & v_s(1,0) &= 5 \\ u_t(1,0) &= 6, & v_t(1,0) &= 4 \\ F_u(2,3) &= -1 & F_v(2,3) &= 10. \end{aligned}$$

Find $W_s(1,0)$ and $W_t(1,0)$.

(2) Solve the following problems. **(Do any two of them)**.

(a) Find the directional derivative of the function $f(x,y) = \sqrt{x+3y} + x^2y^3$ at the point $(3,2)$ in the direction of the vector $\vec{v} = \langle 2, 5 \rangle$.
Hint: Find the unit vector $\hat{v}$ first.

(b) Find the maximum rate of change of $f(x,y,z) = \frac{x+y}{z}$ at the point $(1,1,-1)$ and the direction in which it occurs.

(c) Verify the linear approximation

$$z - e^{x^2-y^2} \approx 2x + 2y - z + 1, \text{at the point } (1,-1,1).$$

(Bonus) Solve the following integrals. **(Do any one of them)**.

(a) If $z = f(x,y)$, where $x = s+t$ and $y = s-t$, show that

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial s}\right)\left(\frac{\partial z}{\partial t}\right).$$

(b) Find the limit

$$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}.$$