Math 1110-01: Practice Final

Early Summer 2024
N.S.

There are 12 questions and you are required to do 10 of them. Each question is worth 10 points. The total score for the exam is 100 points. Doing extra questions will count towards bonus points.

Questions	**Points**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total	

(1) *(10 points)* Solve the following problems:

(a) *(2 points)* A data set consists of the ages at death for each of the 38 past presidents of the United States now deceased.

(i) What is the variable of interest and what type of variable is it?

(ii) Is this set of measurements a population or a sample?

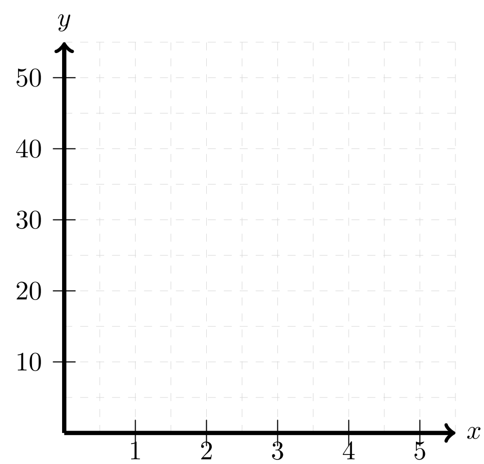
(b) *(2 points)* Fifty people are grouped into four categories— A, B, C, and D—and the number of people who fall into each category is shown in the table:

Category	A	B	C	D
Frequency	11	14	20	5

Construct a Pie chart.

(c) *(3 points)* Construct a line chart for the following data.

Day	1	2	3	4	5
Time (Hours)	55	39	47	15	23



(d) *(3 points)* The following table lists the prices (in euros) of 19 different brands of walking shoes. Construct a stem and leaf plot of the data. Using the plot, determine the distribution of the data i.e. is the data positively skewed, negatively skewed, or symmetrical?

72	57	76	91	54	76	82	50	73	92
90	70	51	65	65	68	89	67	84	73

(2) *(10 points)* Solve the following problem.

(a) *(10 points)* The following data gives bread price for 14 different brands.

0.80	0.83	1.32	0.87	0.89	0.80	0.86
1.91	2.12	1.34	1.05	0.85	0.73	1.16

(i) *(2 points)* Find the mean of the data.

(ii) *(1 point)* Find the median of the data.

(iii) *(1 point)* Find the mode of the data.

(iv) *(2 points)* Find the sample variance and sample standard deviation of the data.

(v) *(1 point)* Find the range of the data.

(vi) *(2 points)* Find the quartiles and the interquartile range of the data.

(vii) *(1 point)* Construct a box plot for the data.

(3) *(10 points)* Solve the following problems:

(a) *(7 points)* A fair six-sided die is tossed twice and the number on the die for each toss is recorded. Let A be the event that the sum of the numbers is 8. Let B be the event that at least one die rolled to 3. Let C be the event that both die rolled to an odd number.

(i) *(3 points)* List the elements of event A, B and C.

(ii) *(2 points)* Find $P(A \cup B)$

(iii) *(2 points)* Find $P(A \mid C')$.

(b) *(2 points)* A French restaurant in Riverside, California, offers a special summer menu in which, for a fixed dinner cost, you can choose from one of two salads, one of two entrees, and one of two desserts. How many different dinners are available?

(c) *(2 points)* A businessmen in New York is preparing an itinerary for a visit to any 6 major cities out of 10 major cities. The distance traveled, and hence the cost of the trip, will depend on the order in which he plans his route. How many different itineraries (and trip costs) are possible?

(4) *(10 points)* Solve the following problems:

(a) *(7 points)* Most coffee drinkers take a little time each day for their favorite beverage, and many take more than one coffee break every day. The table below, adapted from a certain newspaper, shows the probability distribution for x, the number of coffee breaks taken per day by coffee drinkers.

x	3	4	5	6	7	8	9	10	11	12	13
$p(x)$	0.03	0.05	???	0.10	0.14	0.20	0.18	0.12	0.07	0.03	0.01

(i) *(1 point)* Find the probability $P(x = 5)$.

(ii) *(2 points)* Find the mean of the probability distribution x.

(iii) *(2 points)* Find the standard deviation of the probability distribution x.

(iv) *(2 point)* Find $P(4 \leq x < 8 \mid 2 \leq x \leq 11)$.

(b) *(3 points)* Over a long period of time, it has been observed that a professional basketball player can make a free throw on a given trial with probability equal to 0.7. Suppose he shoots 10 free throws.

(i) *(1 point)* What is the probability that he will make at most two free throws?

(ii) *(2 points)* What is the probability that he will make between 3 and 7 (inclusive) free throws?

(5) *(10 points)* Solve the following problems:

(a) *(5 points)* The number of bankruptcies filed in the district court has a Poisson distribution with an average of 7.5 per week.

(i) (1 point) What is the probability that there will be at least 1 bankruptcy filing during a given week?

(ii) *(4 points)* Within what limits does Tchebysheff's Theorem suggest you would expect to see the number of bankruptcy filings per week at least 91.23% of the time?

(b) *(5 points)* A manufacturer of videotapes ships them in lots of 20 videotapes per lot. Before shipment, 9 tapes are randomly selected from each lot and tested. If none is defective, the lot is shipped. If one or more are defective, every tape in the lot is tested. Suppose the lot contains 5 defective videotapes. Let x be the random variable representing defective videotapes.

(i) *(2 points)* Identify the probability distribution for x. Find its mean and variance.

(ii) *(1 point)* What is the probability that the lot will be shipped?

(iii) *(2 points)* What is the probability that at least 4 defective videotapes are selected?

(6) *(10 points)* Solve the following problems:

(a) *(5 points)* Let x have a uniform distribution on the interval 0 to 99. Find the following probabilities:

(i) *(1 points)* $P(x > 4)$.

(ii) *(2 points)* $P(51 \leq x < 72 \,|\, x \geq 12)$.

(iii) *(2 points)* Find the mean and variance of x.

(b) *(5 points)* The length of life (in days) of an alkaline battery has an exponential distribution with an average life of 3 months, so that $\lambda \approx \frac{1}{90}$

(i) *(1 point)* What is the probability that an alkaline battery will fail within 120 days?

(ii) *(3 points)* If a device require two batteries, what is the probability that both batteries last beyond 4 months?

(iii) *(1 point)* Find the mean and standard deviation for the random variable x representing the failure of the battery.

(7) *(10 points)* Solve the following problems:

(a) *(3 points)* Let z be a standard normal random variable with mean $\mu = 4$ and standard deviation $\sigma = 1.5$. Solve the following:

(i) *(1 point)* Find $P(-1.2 \leq x < 1.96)$.

(ii) *(2 points)* Find c such that $P(-c \leq x \leq c) = 0.92$.

(b) *(3 points)* The average length of time required to complete a college achievement test was found to equal 70 minutes with a standard deviation of 12 minutes. When should the test be terminated if you wish to allow sufficient time for 90% of the students to complete the test?

(c) *(4 points)* It is known that 20% of all calls coming into a telephone exchange are long-distance calls. Suppose that 250 calls come into the exchange.

(i) *(2 points)* What is the probability that at least 52 will be long-distance calls?

(ii) *(2 points)* What is the probability that there will be 54 to 57 (inclusive) long-distance calls?

(8) *(10 points)* Solve the following problems:

(a) *(4 points)* Identify the following random sampling:

- *(1 point)* A random sample of public opinion in a small town was obtained by selecting every 10th person who passed by the busiest corner in the downtown area.

- (1 point) One hundred households in each of 10 cities are surveyed concerning average income.

- *(1 point)* One hundred households in Louisiana are surveyed concerning their monthly grocery expenditure.

- *(1 point)* One thousand students are randomly selected from Tulane and asked whether they like snowball or not.

(b) *(4 points)* Allen Shoemaker derived a distribution of human body temperatures with a distinct mound shape. Suppose we assume that the temperature of healthy humans are approximately normal with a mean of 99.° and a standard deviation of 0.97°.

(i) *(2 points)* If 85 healthy people are selected at random, what is the probability that the average temperature of these people is 98.15° or higher.

(ii) *(2 points)* If 100 healthy people are selected at random, what is the probability that the average temperature of these people is between 97.5° and 99.25°.

(c) *(2 points)* A random sample of size $n = 200$ is selected from a binomial distribution with $p = 0.7$. Find the probability that the sample proportion $\hat{p}$ lies between 0.65 and 0.75

(9) *(10 points)* Solve the following problems:

(a) *(3 points)* A random sample of $n = 1500$ observations from a binomial population produced $x = 610$ successes.

(i) *(1 point)* Give the best point estimate for the binomial proportion p.

(ii) *(2 points)* Calculate a 92.5% margin of error.

(b) *(2 points)* A random sample of $n = 100$ measurements has been selected from a population with unknown mean μ and known standard deviation $\sigma = 20$. Calculate the width of the confidence interval for μ for the confidence interval of 87%.

(c) *(3 points)* A supermarket chain packages ground beef using meat trays of two sizes: one that holds approximately 1 pound of meat, and that holds approximately 3 pounds. A random sample of 40 packages in the smaller meat trays produced weight measurements with an average of 1.05 pounds and a standard deviation of 0.22 pounds. Construct a 98.4% confidence interval for the average weight of all packages sold in the smaller meat trays by the supermarket chain.

(d) *(2 points)* As a group, students majoring in engineering disciplines have high salary expectations. A random sample of 60 recent college graduates were selected and the following information was obtained.

Major	**Mean (\$)**	**Standard Deviation**
Electrical Engineering	62,332	12,200
Computer science	57,865	13,430

Let μ_1 and μ_2 represent the population means for electrical engineering major and computer science major respectively. Construct a 94.5% confidence interval for the difference between the means $\mu_1 - \mu_2$.

(10) *(10 points)* Solve the following problems:

(a) *(4 points)* In a study to compare the effects of two pain relievers it was found that of $n_1 = 300$ randomly selected individuals instructed to use the first pain reliever, 186 indicated that it relieved their pain. Of $n_2 = 625$ randomly selected individuals instructed to use the second pain reliever, 432 indicated that it relieved their pain.

(i) *(3 points)* Find a 99% confidence interval for the difference in the proportions experiencing relief from pain for these two pain relievers.

(ii) *(1 point)* Based on the confidence interval in the previous part, is there sufficient evidence to indicate a difference in the proportions experiencing relief for the two pain relievers?

(b) *(2 points)* Independent random samples of $n_1 = n_2 = n$ observations are to be selected from each of two binomial populations 1 and 2. If you wish to estimate the difference in the two population proportions correct to within .05, with probability equal to .96, how large should n be? Assume that $p_1 = p_2$.

(c) *(2 points)* Suppose you wish to estimate a population mean based on a random sample of n observations, and prior experience suggests that $\sigma = 11.5$. If you wish to estimate μ correct to within 1.6, with probability equal to .91, how many observations should be included in your sample?

(d) *(2 points)* Independent random samples of $n_1 = n_2 = n$ observations are to be selected from each of two populations 1 and 2. If you wish to estimate the difference between the two population means correct to within 1.7, with probability equal to 0.95, how large should n_1 and n_2 be? Assume that you know $\sigma_1 = 25.4$ and $\sigma_2 = 21.2$.

(11) *(10 points)* Solve the following problems:

(a) *(5 points)* It is reported that the average or mean number of Facebook friends is 155. Suppose that when 50 randomly chosen Facebook users are polled regarding the number of their friends, the average number of their friends was reported to be $\bar{x} = 152$ with standard deviation of $s = 22.8$. Test the appropriate hypotheses using $\alpha = 0.05$.

(b) *(5 points)* A random sample of $n = 1,000$ observations from binomial population contained 584 successes. You wish to show $p < 0.61$. Calculate the p-value and test it against the confidence coefficient $\alpha = 0.05$.

(12) *(10 points)* Solve the following problems:

(a) *(5 points)* Independent random samples were selected from two quantitative populations, with sample data given below. Using the p-value approach for the data given below, is there sufficient evidence to show that μ_1 is larger than μ_2 at the 2% level of significance?

$$n_1 = 50, \qquad n_2 = 49, \qquad \bar{x}_1 = 36.7, \qquad \bar{x}_2 = 33.5, \qquad s_1 = 4.8, \qquad s_2 = 3.5.$$

(b) *(5 points)* Independent random samples were selected from two binomial populations with sample sizes and the number of success given below:

	Population	
	1	2
Sample Size	900	740
Number of Successes	333	371

Test the hypotheses that $p_1 - p_2 < 0.10$ with $\alpha = 0.07$.

Formula

Confidence Intervals

Parameter	Point Estimator	(1-α)100% Margin Error	(1-α)100% Confidence Interval
μ	$\bar{x}$	$\pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$	$\bar{x} \pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$
p	$\hat{p}$	$\pm z_{\alpha/2}\left(\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$	$\hat{p} \pm z_{\alpha/2}\left(\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$
$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$\pm z_{\alpha/2}\left(\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right)$	$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\left(\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right)$
$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	$\pm z_{\alpha/2}\left(\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$	$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\left(\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$

Sample Size Formulas for Tolerance level B

Parameter	Estimator	Sample Size	Assumptions
μ	$\bar{x}$	$n \geq \frac{z_{\alpha/2}^2\sigma^2}{B^2}$	
$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$n \geq \frac{z_{\alpha/2}^2(\sigma_1^2+\sigma_2^2)}{B^2}$	$n_1 = n_2 = n$
p	$\hat{p}$	$n \geq \frac{z_{\alpha/2}^2\hat{p}\hat{q}}{B^2}$	
$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	$n \geq \frac{z_{\alpha/2}^2(p_1q_1+p_2q_2)}{B^2}$	$n_1 = n_2 = n$

Central Limit Theorem

If random samples of n observations are drawn from a non-normal population with finite mean μ and standard deviation σ, then, when n is large, the sampling distribution of the sample mean $\bar{x}$ is approximately normally distributed, with mean $\mu_{\bar{x}} = \mu$ and standard deviation

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}.$$

For sampling mean of proportion $\hat{p}$, we have

$$\mu_{\hat{p}} = \hat{p}, \qquad \sigma_{\hat{p}} = \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Tchebysheffs' Theorem

Given a number $k \geq 1$ and a set of n measurements then at least $[1 - \frac{1}{k^2}]$ of the measurements will lie within k standard deviations of their mean. *(The theorem applies to every distribution when n is reasonably big).*

$$P(-k\sigma \leq X - \mu \leq k\sigma) \geq 1 - \frac{1}{k^2}.$$