Math 1110-01: Practice Midterm 2

Early Summer 2024
N.S.

Formula

Confidence Intervals

Parameter	Point Estimator	(1-α)100% Margin Error	(1-α)100% Confidence Interval
μ	$\bar{x}$	$\pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$	$\bar{x} \pm z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$
p	$\hat{p}$	$\pm z_{\alpha/2}\left(\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$	$\hat{p} \pm z_{\alpha/2}\left(\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$
$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$\pm z_{\alpha/2}\left(\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right)$	$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\left(\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right)$
$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	$\pm z_{\alpha/2}\left(\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$	$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\left(\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$

Sample Size Formulas for Tolerance level B

Parameter	Estimator	Sample Size	Assumptions
μ	$\bar{x}$	$n \geq \frac{z_{\alpha/2}^2\sigma^2}{B^2}$	
$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$n \geq \frac{z_{\alpha/2}^2(\sigma_1^2+\sigma_2^2)}{B^2}$	$n_1 = n_2 = n$
p	$\hat{p}$	$n \geq \frac{z_{\alpha/2}^2\hat{p}\hat{q}}{B^2}$	
$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	$n \geq \frac{z_{\alpha/2}^2(p_1q_1+p_2q_2)}{B^2}$	$n_1 = n_2 = n$

Central Limit Theorem

If random samples of n observations are drawn from a non-normal population with finite mean μ and standard deviation σ, then, when n is large, the sampling distribution of the sample mean $\bar{x}$ is approximately normally distributed, with mean $\mu_{\bar{x}} = \mu$ and standard deviation

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}.$$

For sampling mean of proportion $\hat{p}$, we have

$$\mu_{\hat{p}} = \hat{p}, \qquad \sigma_{\hat{p}} = \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

(1) Solve the following problem:

(a) The table below describes the probability distribution for the discrete random variable x.

x	0	1	2	3	4
$p(x)$	0.11	0.3	0.21	???	0.15

(i) Find the mean of x.

(ii) Find the variance of x.

(iii) Find $P(1 < x \leq 2)$.

(iv) Find $P(x \geq 1 \mid x < 4)$

(b) Consider a binomial random variable with $n = 9$ and $p = 0.7$. Let x be the number of successes in the sample. Evaluate the following probabilities:

(i) $P(2 < x \leq 7)$.

(ii) $P(x > 4)$.

(iii) Find $P(x \geq 3 \mid 1 \leq x \leq 7)$.

(c) The number of visits to a website is known to have a Poisson distribution with a mean of 7 visits per minute.

(i) What is the probability that the number of visits is less than or equal to 11?

(ii) What is the probability that the number of visits is between 3 and 10 (exclusive)?

(iii) What is the probability that the number of visits is greater than or equal to 12?

(d) A company ships a slot of 12 televisions which contains 3 defectives. Suppose the customer randomly selects 5 televisions and tests them. Find the following probabilities:

(i) The customer selects no defectives.

(ii) The customer selects 2 defectives

(iii) The customer selects at least 1 defective

(iv) The customer selects at most 2 defectives.

(2) Solve the following problems:

(a) (6 points) Let x have a uniform distribution on the interval -5 to 20. Find the following probabilities:

(i) $P(x \leq 3)$.

(ii) $P(x \geq 4)$.

(iii) $P(-1 < x < 11)$.

(b) Let x have an exponential distribution with probability $\lambda = 0.4$. Find the following probabilities:

(i) $P(x < 4)$.

(ii) $P(x \geq 5)$.

(iii) $P(0.5 < x < 11 \,|\, x < 5)$.

(c) (5 points) Let z be a standard normal random variable with mean $\mu = 0$ and standard deviation $\sigma = 1$. Solve the following:

(i) Find $P(z \geq 1.27)$.

(ii) Find $P(-1.55 \leq z < 1.99)$.

(iii) Find c such that $P(z \geq c) = 0.64$.

(iv) Find c such that $P(-c \leq z \leq c) = 0.96$.

(3) Solve the following problems:

(a) The diameter x of Douglas fire grown at a Christmas tree farm are normally distributed with a mean of 5 inches and a standard deviation of 1.5 inches.

(i) What is the probability that the trees will have diameters between 4 and 8 inches?

(ii) What is the probability that the trees will have diameters less than 7.5 inches?

(iii) Suppose that your Christmas tree stand expands to a diameter of 9 inches. What is the probability of the tree that it will not fit in your Christmas tree stand?

(iv) Find the diameter c such that $P(x \geq c) = 0.02$.

(b) Consider a binomial random variable x with $n = 150$ and $p = 0.3$.

(i) Find the mean for the binomial random variable x.

(ii) Find the standard deviation for the binomial random variable x.

(iii) Use the correction for continuity and approximate $P(43 \leq x < 49)$ using the normal approximation.

(4) Solve the following problems:

(a) Identify the following random sampling:

- A random sample of public opinion in Athens was obtained by selecting every 25th person who passed by the busiest corner in the downtown area.

- One hundred households in each of 10 countries are surveyed concerning travel expenditures.

- One hundred households in California are surveyed concerning their annual income.

- One thousand students are randomly selected from LSU and asked whether they like snowball or not.

(b) Suppose a random sample of $n = 124$ observations is selected from a population that is normally distributed with mean equal to 110 and standard deviation equal to 14.2.

(i) Find the mean and the standard deviation of the sampling distribution of the sample mean $\bar{x}$.

(ii) Find the probability $P(\bar{x} \geq 115)$.

(iii) Find the probability $P(103 < \bar{x} \leq 112)$.

(c) A random sample of $n = 200$ is selected from a binomial distribution x‘with $p = 0.2$.

(i) Calculate the mean and standard deviation of the sampling distribution of $\hat{p}$.

(ii) Find the probability that the sample proportion $\hat{p}$ is greater than 0.23.

(iii) Find the probability that the sample proportion $\hat{p}$ lies between 0.24 and 0.26.

(5) Solve the following problems:

(a) Find the z-value needed to calculate large sample confidence intervals for the for the following confident coefficient $1 - \alpha = 0.91$. That is find $z_{\alpha/2}$.

(b) Do our children spend enough time enjoying the outdoors and playing with friends, or are they spending more time glued to the television, computer and their cell phones? A random sample of 250 youth between he ages of 8 and 18 showed that 160 of them had a TV in their bedroom and that 130 had a video game console in their bedroom.

(i) Estimate the proportion of all 8-18 years olds who have a TV in their bedroom.

(ii) Calculate the 95% margin of error for your estimate.

(iii) Estimate the proportion of all 8-18 years olds who have a video game console in their bedroom.

(iv) Calculate the 90% margin of error for your estimate.

(c) A survey is designed to estimate the proportion of SUV being driven in the state of California. A random sample of 1000 registration selected from a DMV database, and 69 are classified as SUVs.

(i) Give a 94% confidence interval to estimate the proportion of SUV in California.

(ii) Give a 93% **upper confidence bound** to estimate the proportion of SUV in California.

(iii) How can you estimate the proportion of sports utility vehicles in California with a higher degree of accuracy?

Bonus (15 points) Solve the following problems:

(a) (5 points) Independent random samples were selected from two populations with sample sizes, means, and variances are given below:

	Population	
Sample Size	47	58
Sample mean	10.3	8.3
Sample Variance	9.89	15.42

Construct a 94% confidence interval for the difference in mean scores $(\mu_1 - \mu_2)$ for the two teaching methods.

(b) (5 points) Independent random samples were selected from two binomial populations, with sample sizes and the number of successes given below

$$n_1 = 1000, \qquad n_2 = 1100, \qquad x_1 = 400, \qquad x_2 = 495.$$

Construct a 96% confidence interval for the difference between population proportions $(p_1 - p_2)$.

(c) Suppose that you were designing a research poll that included questions about illegal immigration into the United States, and the federal and state responses to the problem. If you wanted to estimate the percentage of the population who agree with a particular statement in your survey questionnaire correct to within 3% with probability 0.95, approximately what minimum number of people would have to be polled? Given that $p = 0.51$.