Math 2211: Practice Midterm 3

Calculus 3

(1) Solve the following problems.

(a) Find the local maximum and minimum values and saddle points of the function

$$f(x, y) = e^y(y^2 - x^2)$$

(b) Show that $f(x, y) = x^2 + 4y^2 - 4xy + 2$ has an infinite number of critical points and $f_{xx}f_{yy} - f_{xy}^2$ evaluated at each of the critical point is 0.

(c) Find the absolute maximum and minimum values of $f(x, y) = 2x^3 + y^4$ on $D = \{(x, y) \mid x^2 + y^2 \leq 1\}$.

(d) Find the absolute maximum and minimum values of $f(x, y) = 3 + xy - x - 2y$ on the closed triangle D with vertices $(1, 0), (5, 0)$ and $(1, 4)$.

(e) Find the absolute maximum and minimum values of $f(x, y) = xy^2$ on $D = \{(x, y) \mid x \geq 0, y \geq 0, x^2 + y^2 \leq 3\}$.

(2) Solve the following:

(a) Find the points on the surface $y^2 = 9 + xz$ that are closest to the origin.

(b) Find three positive integer whose sum is 100 and whose product is maximum.

(c) Use Langrange multipliers to find the maximum and minimum values of the function $f(x, y) = xyz$ subjected to the constraint $x^2 + 2y^2 + 3z^2 = 6$.

(d) Find the extreme values of $f = 2x^2 + 3y^2 - 4x - 5$ on the region described by the inequality $x^2 + y^2 \leq 16$.

(e) Use Lagrange multipliers to prove that the rectangle with maximum area that has a given perimeter p is a square.

(3) Solve the following problems.

(a) Evaluate the integral

$$\iint_R \sqrt{1-x^2}\, dA, \quad \text{where } R = \{(x,y) | -1 \le x \le 1,\ -2 \le y \le 2\}\ .$$

(b) Evaluate the integral

$$\iint_R (3x+6y)^5\, dA, \quad \text{where } R = [0,1] \times [0,2]\ .$$

(c) Evaluate the integral

$$\iint_R xy^2\sqrt{x^2+y^3}\, dA, \quad \text{where } R = [0,1] \times [0,1]\ .$$

(d) Evaluate the integral

$$\iint_R xye^{x^2 y}\, dA, \quad \text{where } R = [0,1] \times [0,\pi]\ .$$

(e) Evaluate the integral

$$\iint_R \frac{1+x^2}{1+y^2}\, dA, \quad \text{where } R = [0,1] \times [0,1] \ .$$

(f) Find the volume of the solid that lies under the hyperbolic paraboloid $z = 4 + x^2 - y^2$ and above the square $R = [-1,1] \times [0,2]$.

(g) If $f(x,y)$ is continuous on $[a,b] \times [c,d]$ and

$$g(x,y) = \int_a^x \int_b^y f(s,t)\; dt\, ds$$

for $a < x < b$, $c < y < d$, show that $g_{xy} = g_{yx} = f(x,y)$.

(4) Solve the following problems:

(a) Evaluate the integral

$$\iint_D x \cos y \, dA, \quad \text{where } D \text{ is bounded by } y = 0,\ y = x^2,\ x = 1\ .$$

(b) Evaluate the integral

$$\iint_D x^3 \, dA, \quad \text{where } D = \{(x, y) |\, 1 \le x \le e,\ 0 \le y \le \ln x\}\ .$$

(c) Evaluate the integral

$$\iint_D (x + y) \, dA, \quad \text{where } D \text{ is bounded by } y = \sqrt{x},\ y = x^2\ .$$

(d) Evaluate the integral

$$\iint_D y^3 \, dA, \quad \text{where } D \text{ is the triangular region with vertices } (0, 2), (1, 1), (3, 2)\ .$$

(e) Evaluate the integral

$$\iint_D (2x - y)\, dA, \quad \text{where } D \text{ is bounded by the circle with center the origin and radius 2 .}$$

(f) Show that the volume of a sphere with radius a is $\frac{4}{3}\pi r^3$

(g) Sketch and change the order of integration.

$$\int_0^4 \int_0^{\sqrt{x}} f(x, y)dydx$$

(h) Sketch and change the order of integration.

$$\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x, y)dxdy$$

(i) Change the order of integration.

$$\int_0^1 \int_{\arctan x}^{\pi/4} f(x, y)dydx$$

(5) Solve the following:

(a) Evaluate

$$\iint_D xy\, dA, \text{ where } D \text{ is the disk with center the origin and radius 3.}$$

(b) Evaluate

$$\iint_D \cos\left(x^2+y^2\right) dA, \text{ where } D \text{ is the region that lies above the } x\text{-axis within the circle } x^2+y^2=9.$$

(c) Evaluate

$$\iint_D e^{-x^2-y^2}\, dA, \text{ where } D \text{ is the region bounded by the semicircle } x=\sqrt{4-y^2} \text{ and the } y\text{-axis.}$$

(d) Evaluate

$$\iint_R \arctan(y/x)\, dA, \text{ where } R = \{(x,y)\,|\,1 \leq x^2 + y^2 \leq 4,\ 0 \leq y \leq x\}$$

(e) Use polar coordinates to find the volume of the solid under the cone $z = \sqrt{x^2 + y^2}$ and above the disk $x^2 + y^2 \leq 4$.

(f) Use polar coordinates to find the volume of a sphere of radius a.

(g) Use a double integral to find the area of the region within both of the circles $r = \cos\theta$ and $r = \sin\theta$.