Math 2211: Recitation 11 (T)

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(1) Do all **three** the following problems:

(a) Evaluate the line integral $\int_C xy\,ds$ where $C : x = t^2,\ y = 2t,\ \ 0 \leq t \leq 1$.

(b) Evaluate the line integral $\int_C xe^y\,dx$ where C is the arc of the curve $x = e^y$ from $(1, 0)$ to $(e, 1)$.

(c) Let $\mathbf{F}(x, y, z) = \langle \sin x, \cos y, xz \rangle$ and $\mathbf{r}(t) = \langle t^3, -t^2, t \rangle$ where $0 \leq t \leq 1$. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is given by the vector function $\mathbf{r}(t)$.

(2) Solve the following problems. **(Do any two of them)**.

(a) Use the given transformation to evaluate the integral $\iint_R (x - 6y)dA$, where R is the triangular region with vertices $(0, 0), (5, 1)$ and $(1, 5)$; with transformation $x = 5u + v,\ y = u + 5v$.

(b) Let $\mathbf{F}(x,y) = \langle xy^2, x^2y \rangle$ and C : $\mathbf{r}(t) = \langle t + \sin(\frac{\pi t}{2}), t + \cos(\frac{\pi t}{2}) \rangle$ where $t \in [0,1]$. Find a function f such that $\mathbf{F} = \nabla f$ and use it to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(c) Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

$$\int_C xy^2\,dx + 2x^2y\,dy, \quad C \text{ is the triangle with vertices (0,0),(2,2) and (2,4).}$$

(Bonus) Solve the following integrals. **(Do any one of them)**.

(a) Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

$$\int_C (y + e^{\sqrt{x}})dx + (2x + \cos(y^2))dy, \quad C \text{ is the boundary of the region enclosed by } y = x^2 \text{ and } x = y^2.$$

(b) Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F} = \langle e^x + x^2y\,,\, e^y - xy^2 \rangle, \quad C \text{ is the circle } x^2 + y^2 = 25 \text{ oriented clockwise.}$$